

Confidence Intervals and Hypothesis Tests: Two Samples

9.1 Z-Intervals to Compare Two Population Means: Independent Samples

1. The University of Texas at Austin conducted research to look at the relationship of ethnicity and gender to the age of sexual debut. One variable of interest was the age at which the participants first had intercourse. A group of 123 Hispanic women reported an average age of 16.52 years and a standard deviation of 2.25 years. A group of 123 Asian women reported an average age of 17.63 years with a standard deviation of 2.78 years. Find the 95% confidence interval for the true difference between the age at first intercourse for Hispanic women and Asian women.
2. A randomly selected sample of 53 males who took the SAT in 2008 scored an average of 533 points on the math portion of the exam. Their standard deviation was 118 points. A random selection of 55 females from the same year had an average score of 500 and a standard deviation of 111. Using this sample data, construct a 98% confidence interval for the true difference between the average scores of males and females on the SAT math portion of the exam. Is there a significant difference between the sexes?
3. Researchers from Syracuse University and the University of Rochester in NY surveyed 481 women who visited a publicly funded STD clinic. They were conducting research on the effects of childhood sexual abuse (CSA) on sexual risk behavior. After having each woman complete a computer survey about their current and childhood sexual experiences, they divided the results into two categories. One category for the women who had suffered CSA, and another category for those who had not experienced CSA. One of the questions in the survey asked about the total number of sexual partners in their lifetime. For the 275 women who had not experienced CSA, the mean number of partners was 16.7 with a standard deviation of 18.7. For the 206 women who had experienced CSA, the mean number of partners was 29.8 with a standard deviation of 26.3. Form a 90% confidence interval for the true average difference between the number of sexual partners for women who have no experience with CSA and for women who have suffered CSA. The women in this survey were visiting an STD clinic. Does this have an effect on the ability to generalize the results found in the study?

4. The following interval was created to compare the average time (in minutes) spent on social networking sites like Facebook and Twitter during final's week and during spring break: $-38.9 < \mu_{finals} - \mu_{spring} < -19.1$. Does the interval indicate that more time is spent on social networking sites during spring break or less time?
5. A confidence interval was created to compare the fuel economy (in miles per gallon) of current year Ford truck models and Ford Truck models from 2006. Interpret the interval that resulted from that comparison: $0.3 < \mu_{current} - \mu_{2006} < 2.1$.

Answers:

$$1. \quad z_{\alpha/2} = 1.960, E = 1.960 \sqrt{\frac{2.25^2}{123} + \frac{2.78^2}{123}} \approx 0.63205$$

$$[(16.52 - 17.63) - E, (16.52 - 17.63) + E]$$

We are 95% confident the true mean difference is inside the following interval: $[-1.74, -0.48]$.

$$2. \quad z_{\alpha/2} = 2.326, E = 2.326 \sqrt{\frac{118^2}{53} + \frac{111^2}{55}} \approx 51.3164$$

$$[(533 - 500) - E, (533 - 500) + E]$$

We are 98% confident the true mean difference is inside the following interval: $[-18.32, 84.32]$. The interval does not show a significant difference since it is possible the true difference is zero (this is because zero is found inside the interval).

$$3. \quad z_{\alpha/2} = 1.645, E = 1.645 \sqrt{\frac{18.7^2}{275} + \frac{26.3^2}{206}} \approx 3.5394$$

$$[(16.7 - 29.8) - E, (16.7 - 29.8) + E]$$

We are 90% confident the true mean difference is inside the following interval: $[-16.64, -9.561]$. The fact that these women were visiting a free STD clinic does affect the validity of the interval. These women may be different from the typical women in the larger population. It could be that they are generally more promiscuous than the rest of the population, so this data may not be representative of the population.

4. Since both limits of the interval are negative, we can assume the second mean is larger than the first. Since we subtracted the mean for spring from the mean for finals week, the mean for spring is the second mean, so it is larger.

5. Since both limits of the interval are positive, we can assume the first mean is larger than the second. Since we subtracted the mean for 2006 models from the mean for current models, the mean for current year models is the first mean, so it is larger.